یوسف قاسمی

۲ — ①

$$\frac{a}{q},\ a,\ aq \longrightarrow \frac{4a}{q},\ 8a,\ 16aq$$

$$2(8a) = \frac{4a}{q} + 16aq \xrightarrow{a \neq 0} 4 = \frac{1}{q} + 4q \rightarrow 4q^2 - 4q + 1 = 0$$

$$\boxed{q = \frac{1}{2}}$$

$$\left(\frac{a}{q}\right)^2 + a^2 + (aq)^2 = \frac{4a}{q} + 8a + 16aq \xrightarrow{q=\frac{1}{2}} a = \frac{4 \times 8}{7}$$

جمله اول $= \frac{a}{q} = \frac{4 \times 8}{7} \times 2 = \frac{64}{7}$

یوسف قاسمی

$۴ \leftarrow \boxed{۲}$

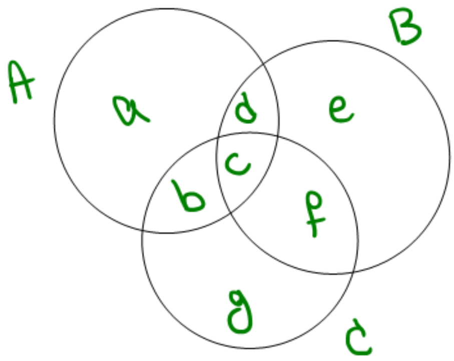
$$y = kx^{۲} - ۴x - ۶$$

$$S \begin{cases} x = -\dfrac{-۴}{۲k} = \dfrac{۲}{k} \\ y = -\dfrac{۱۶ + ۴ \times ۶k}{۴k} = -\dfrac{۴ + ۶k}{k} \end{cases}$$

$$\xrightarrow{y = -۴x - ۴} -\dfrac{۴ + ۶k}{k} = -۴ \times \dfrac{۲}{k} - ۴$$

$$\boxed{k = ۲} \longrightarrow y = -\dfrac{۱۶}{۲} = -۸$$

۱ ← $\boxed{۳}$



$$(A-B)' = (A\cap B')' = A'\cap B$$

$$= \{e, f\}$$

$$B - C = \{e, d\}$$

$$\big((A'\cap B) - (B-C)\big) - C = \{f\} - \{b, c, f, g\} = \varnothing$$

$$A' = \{e, f, g\} \qquad B\cup C = \{b, c, d, e, f, g\}$$

$$A' - (B\cup C) = \varnothing$$

روش دوم:

$$(A\cap B')' \cap (B\cap C')' \cap C'$$

$$= (A'\cup B)\cap(C\cup B')\cap C'$$

$$= (A'\cup B)\cap(\underbrace{C\cap C'}_{\varnothing})\cap(B'\cap C') = (B\cup A')\cap(B'\cap C')$$

$$= \underbrace{(\underbrace{B\cap B'\cap C'}_{\varnothing})}_{\varnothing} \cup (A'\cap B'\cap C')$$

$$= A'\cap(B\cup C)'$$

$$= A' - (B\cup C)$$

یوسف قاسمی

۲ ← ۴

$$(r \wedge (p \vee q)) \vee (\sim(p \vee q) \wedge r)$$

$$= r \wedge \big((p \vee q) \vee (\sim(p \vee q))\big) = r$$

$$-\begin{cases} x^{۲}+۴x+m=۰ \\ x^{۲}+۲x-۳m=۰ \end{cases} \longrightarrow ۴x+۴m=۰ \longrightarrow x=-m$$

$$m^{۲}-۵m=۰ \longrightarrow m=۰,\ \boxed{۵}\ \checkmark$$

$$۳ \longleftarrow \boxed{۵}$$

$$x^{۲}+۲x-۱۵=۰$$

$$(x-۳)(x+۵)=۰ \longrightarrow \boxed{x=۳,-۵}$$

$$x^{۲}+۴x+۵=۰$$

$$x=-۱,-۵$$

اختلاف $=۳-(-۱)=۴$

$۴ \leftarrow \boxed{۶}$

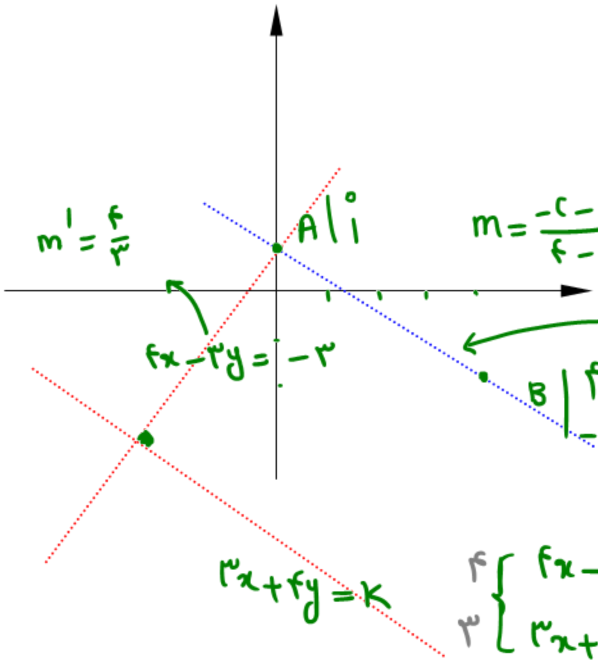
$$-۲ < \frac{۲}{x^۲-۳x+۲} < ۰ \rightarrow -۱ < \frac{۱}{x^۲-۳x+۲} < ۰$$

همواره + $\qquad \Delta = ۹-۱۲ < ۰$

$$\left(\frac{۱}{x^۲-۳x+۲}+۱\right)\left(\frac{۱}{x^۲-۳x+۲}\right) < ۰ \rightarrow \frac{x^۲-۳x+۳}{\underbrace{(x^۲-۳x+۲)^۲}_{+}} < ۰$$

$\varnothing$

$\boxed{7} \rightarrow 3$



$m' = \frac{4}{3}$

$A \left| \begin{matrix} 0 \\ 1 \end{matrix} \right.$

$m = \frac{-2-1}{4-0} = \frac{3}{-4}$

$3x + 4y = 4$

$4x - 3y = -3$

$B \left| \begin{matrix} 4 \\ -2 \end{matrix} \right.$

$AB = 5$

A فاصله تا خط $= 5 = \frac{|4-k|}{\sqrt{9+16}}$

$|k-4| = 25$

$k - 4 = \pm 25$

$\begin{cases} k = 21 \rightarrow \\ k = -29 \end{cases}$

$3x + 4y = k$

$\begin{array}{r} 4 \\ 3 \end{array} \begin{cases} 4x - 3y = -3 \\ 3x + 4y = 21 \end{cases}$

$25x = -75 \rightarrow \boxed{x = -3}$

یوسف قاسمی

$f(x) = \log(2x-5)$ $\qquad$ $g(x) = x + \sqrt{2x-4}$ $\qquad$ $2 \leftarrow \boxed{8}$

$$0 \longrightarrow \boxed{f^{-1}} \longrightarrow \boxed{g^{-1}} \longrightarrow \alpha$$

$$0 \longleftarrow \boxed{f} \xleftarrow{3} \boxed{g} \longleftarrow$$

$$\log(\overbrace{2x-5}^{1}) = 0$$

$$x = 3$$

$$x + \sqrt{2x-4} = 3$$

$$2 \leqslant x \leqslant 3 \qquad \sqrt{2x-4} = 3 - x$$

$$2x - 4 = 9 - 6x + x^2$$

$$x^2 - 8x + 13 = 0$$

$$\Delta' = 16 - 13 = 3$$

$$x = 4 \cancel{\pm} \sqrt{3}$$

$\boxed{۹}$

$$f(x) = ۲ + ۲^{b-ax}$$

$$g(x) = -x^۲ - ۳x + ۸$$

$$\xrightarrow{x=۱} ۲ + ۲^{b-a} = -۱ - ۳ + ۸$$

$$۲^{b-a} = ۲^۱ \rightarrow \boxed{b-a=۱}$$

$$۱۰ \rightarrow \boxed{f^{-۱}} \rightarrow -۱$$

$$۱۰ \leftarrow \boxed{f} \leftarrow -۱$$

$$f(-۱) = ۲ + ۲^{b+a} = ۱۰$$

$$۲^{b+a} = ۸ = ۲^۳$$

$$\boxed{b+a=۳}$$

$$\begin{cases} b-a=۱ \\ b+a=۳ \end{cases} \rightarrow b=۲, a=۱$$

$$۲b - a = ۴ - ۱ = ۳$$

$4 \leftarrow \boxed{10}$

$$\frac{1}{x+2} - \frac{x^2-9x-2}{(x+2)(x^2-2x+4)} = \frac{6x}{x^2-2x+4}$$

$$x^2-2x+4-x^2+9x+2 = 6x(x+2)$$

$$7x+6 = 6x^2+12x$$

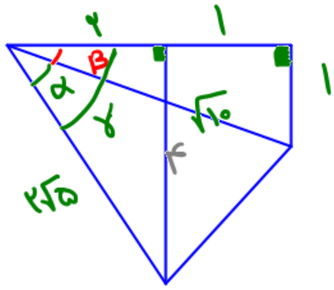
$$6x^2+5x-6=0$$

$$\Delta = 25+4\times 36 = 25+144 = 169$$

$$x = \frac{-5\pm 13}{12} \quad \begin{cases} + \;\; x = \frac{8}{12} = \frac{2}{3} \\ - \end{cases}$$

یوسف قاسمی

$\boxed{11} \rightarrow ۲$



$\cos\alpha = ?$

$\cos\beta = \frac{3}{\sqrt{10}} \quad \sin\beta = \frac{1}{\sqrt{10}}$

$\alpha + \beta = \gamma$

$\alpha = \gamma - \beta \rightarrow \cos\alpha = \cos(\gamma - \beta) = \cos\gamma\cos\beta + \sin\gamma\sin\beta$

$= \frac{2}{2\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{4}{2\sqrt{5}} \times \frac{1}{\sqrt{10}}$

$= \frac{2\times 3 + 4}{2\sqrt{5}\times\sqrt{10}} = \frac{\cancel{2}\times 5 = (\sqrt{5})^2}{\cancel{2}\sqrt{5}\sqrt{10}} = \frac{\sqrt{5}}{\sqrt{10} = \sqrt{5}\times\sqrt{2}}$

$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

يوسف قاسمی

$\boxed{12} \rightarrow 3$

$$y = a + b\cos\left(cx - \frac{\pi}{2}\right)$$

$$T = \frac{4\pi}{3} - \left(-\frac{2\pi}{3}\right) = 2\pi = \frac{2\pi}{|c|} \rightarrow |c| = 1 \xrightarrow{c>0} c = 1$$

$$\left.\begin{array}{l} y_{max} = a + |b| = 1 \xrightarrow{b>0} a + b = 1 \\ f(0) = 0 \rightarrow a + \frac{b}{2} = 0 \rightarrow 2a + b = 0 \end{array}\right\} \begin{array}{l} a = -1 , b = 2 \\ \Rightarrow b(c - a) = 2\left(1 - (-1)\right) = 4 \end{array}$$

$4 \leftarrow \boxed{13}$

$$\cos\left(\frac{17\pi}{8}+x\right)=\cos\left(\frac{16\pi+\pi}{8}+x\right)=\cos\left(2\pi+\frac{\pi}{8}+x\right)$$
$$=\cos\left(\frac{\pi}{8}+x\right)$$

$$\cos\left(\frac{3\pi}{8}-x\right)=\cos\left(\frac{4\pi-\pi}{8}-x\right)=\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{8}+x\right)\right)=\sin\left(\frac{\pi}{8}+x\right)$$

$$\cos\left(\frac{\pi}{8}+x\right)\sin\left(\frac{\pi}{8}+x\right)=\left(\frac{1}{2}\right)^2$$

$$2\sin\left(\frac{\pi}{8}+x\right)\cos\left(\frac{\pi}{8}+x\right)=\frac{1}{2}$$

$$\sin\left(\frac{\pi}{4}+2x\right)=\frac{1}{2}$$

$$\frac{\pi}{4}+2x=2k\pi+\frac{\pi}{6} \qquad \frac{\pi}{4}+2x=2k\pi+\frac{5\pi}{6}$$

$$x=\frac{(24k-1)\pi}{24} \qquad x=\frac{(24k+7)\pi}{24}$$

$$\left[-\frac{12\pi}{24},\frac{12\pi}{24}\right]$$

k	x
0	$-\frac{\pi}{24}$

k	x
0	$\frac{7\pi}{24}$

$$\xrightarrow{\text{مجموع}} \frac{6\pi}{24}=\frac{\pi}{4}$$

$$f(x) = \frac{|x-۲|}{۲} \to \frac{|x-۲|}{۲} = ۲ \to |x-۲| = ۴ \to x-۲ = \pm ۴$$

$$x = ۶ \,,\, -۲$$

$$۲ \leftarrow \boxed{۱۴}$$

$$x+۲ \longrightarrow \boxed{g} \xrightarrow[-۲]{۶} \boxed{f} \xrightarrow{۲} \boxed{g} \longrightarrow ۰$$

$\boxed{15} \rightarrow 1$

معادله‌ی خط $\frac{x}{m}+\frac{y}{\pi}=1 \rightarrow y=-\frac{\pi}{m}x+\pi=f^{-1}(x)$

$m<0$

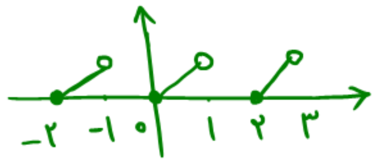
$$f(x)=-\frac{m}{\pi}x+m$$

$$\lim \frac{-\frac{\pi}{m}x+\cdots}{-\frac{m}{\pi}x+\cdots}=\frac{\pi^2}{m^2}=\pi \rightarrow m^2=\pi$$

$$m=-\sqrt{\pi}$$

$\boxed{14} \rightarrow 1$

$y = x - [x]$ ، $0 \leq \cdots < 1$ زوج $[x]$



$y = |x - [x-a]|$ فرد $[x]$ $\Rightarrow y = [x-a] - x$

با توجه به نمودار برای پیوستگی

حد تابع در نقاط زوج باید صفر باشد.

$$\lim_{x \to (2k)^-} [x-a] - x = 0 \rightarrow [(2k)^- - a] - 2k = 0$$

$$[2k - a] = 2k$$

$$\xrightarrow{\text{تعریف}} 2k \leq 2k - a < 2k + 1$$

$$\xrightarrow{k=0} 0 \leq -a < 1 \longrightarrow -1 < a \leq 0$$

طبق فرض $a < -1$

در نتیجه مقداری برای a وجود ندارد.

$\boxed{۱۷} \longrightarrow ۲$

$$f(x) = \frac{x}{1 - x|x|} \qquad D_f = \mathbb{R} - \{1\}$$

$$x|x| = 1 \begin{cases} \xrightarrow{x \geq 0} x^2 = 1 \rightarrow x = 1 \\ \xrightarrow{x < 0} -x^2 = 1 \quad \times \end{cases}$$

$$f(x) = \begin{cases} \dfrac{x}{1-x^2} & x \geq 0 \\ \dfrac{x}{1+x^2} & x < 0 \end{cases} \longrightarrow f'(x) = \begin{cases} \dfrac{1 - x^2 + 2x^2}{(1-x^2)^2} & x \geq 0 \\ \dfrac{1 + x^2 - 2x^2}{(1+x^2)^2} & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{1+x^2}{(1-x^2)^2} & x \geq 0 \\ \dfrac{1-x^2}{(1+x^2)^2} & x < 0 \end{cases}$$

طول نقاط بحرانی

$$f'(x) = 0 \;\Big\langle\; 1 - x^2 = 0 \xrightarrow{x < 0} \boxed{x = -1}$$

$3 \leftarrow \boxed{18}$

$$f(x)=|4x-3|\sqrt{ax}$$

$$f'(x)=\frac{4(4x-3)}{|4x-3|}\sqrt{ax}+\frac{a}{2\sqrt{ax}}|4x-3|=\begin{cases}4\sqrt{ax}+\dfrac{a(4x-3)}{2\sqrt{ax}} & x>\dfrac{3}{4}\\ -(\quad\quad) & x<\dfrac{3}{4}\end{cases}$$

$$f'_{+}\left(\frac{3}{4}\right)=4\sqrt{a\times\frac{3}{4}}=2\sqrt{3a} \longrightarrow 4\sqrt{3a}=2\sqrt{6}$$

$$f'_{-}\left(\frac{3}{4}\right)=-2\sqrt{3a}$$

$$2\sqrt{3a}=\sqrt{6}$$

$$4\times 3a=6 \longrightarrow \boxed{a=\frac{1}{2}}$$

$\boxed{19}$ سوال غلط

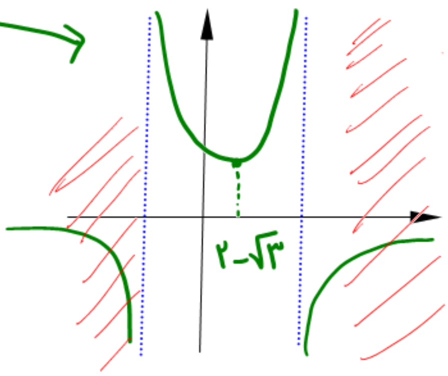
$$f(x)=(m^2-1)x^2+(2-m)x+5$$

α , β ریشه‌های معادله هستند.

$$\Delta>0 \to (m-2)^2-4(m^2-1)5>0 \to \frac{-2-2\sqrt{115}}{19}<m<\frac{-2+\sqrt{115}}{19}$$

$$\alpha+\beta=-\frac{b}{a}=\frac{m-2}{m^2-1} \to f'(m)=\frac{m^2-1-2m(m-2)}{(m^2-1)^2}=0 \to m=2\pm\sqrt{3}$$

نمودار
$$f(m)=\frac{m-2}{m^2-1}$$



$$m=\frac{-2-2\sqrt{115}}{19} \qquad m=\frac{-2+2\sqrt{115}}{19}$$